

# Partial Dynamical $SU(3)$ Symmetry in Deformed Nuclei

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**Abstract.** We discuss characteristic features of  $SU(3)$  partial dynamical symmetry in relation to nuclear spectroscopy and compare with previous broken- $SU(3)$  calculations for  $^{168}\text{Er}$ .

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Elliott's pioneering work [1] established  $SU(3)$  as a relevant symmetry for axially-deformed nuclei. In the Interacting Boson Model (IBM) [2] of nuclei, such a symmetry appears as a dynamical symmetry in which rotational bands are arranged into irreducible representations (irreps),  $(\lambda, \mu)$ , of  $SU(3)$ . For  $N$  bosons the ground band  $g(K=0)$  spans the irrep  $(2N, 0)$  while the  $\beta(K=0)$  and  $\gamma(K=2)$  bands are degenerate and span the irrep  $(2N-4, 2)$ . The undesired  $\beta$ - $\gamma$  degeneracy can be lifted by adding an  $O(6)$  term to the  $SU(3)$  Hamiltonian, as done by Warner, Casten and Davidson (WCD) [3] or by considering a general (non- $SU(3)$ ) quadrupole operator, as in the consistent-Q formalism (CQF) [4]. In these procedures the  $SU(3)$  symmetry is completely broken, all eigenstates are mixed and none of the virtues of an exact dynamical symmetry (*e.g.* solvability) are retained. In contrast, the recently introduced partial dynamical symmetry (PDS) [5] corresponds to a particular breaking of  $SU(3)$ , such that **part** of the states (but not all) are still solvable with good symmetry.

The work of [5] demonstrated the relevance of  $SU(3)$  PDS to the spectroscopy of  $^{168}\text{Er}$ . The Hamiltonian used has the form

$$H = h_0 P_0^\dagger P_0 + h_2 P_2^\dagger \cdot \tilde{P}_2 + \lambda \hat{C}_{O(3)} \quad . \quad (1)$$

where  $P_0^\dagger = d^\dagger \cdot d^\dagger - 2(s^\dagger)^2$ ,  $P_{2,\mu}^\dagger = 2s^\dagger d_\mu^\dagger + \sqrt{7}(d^\dagger d^\dagger)_\mu^{(2)}$  are IBM boson-pairs and  $\hat{C}_{O(3)}$  the Casimir operator of  $O(3)$ . For  $h_0 = h_2$  the first two terms of Eq. (1) form an  $SU(3)$  scalar related to the Casimir operator of  $SU(3)$ , while for  $h_0 = -5h_2$  they form an  $SU(3)$  tensor,  $(\lambda, \mu) = (2, 2)$ . Although the Hamiltonian is not an  $SU(3)$  scalar, it has a subset of **solvable** states with good  $SU(3)$  symmetry. The  $O(3)$  rotational term is diagonal, contributes an  $L(L+1)$  splitting and has no effect on wave functions. The solvable eigenstates of  $H$  belong to the ground and  $\gamma_{K=2k}^k$  bands and are simply selected members of the Elliott basis [1] with good  $SU(3)$  symmetry,  $(\lambda, \mu) = (2N-4k, 2k)K=2k$ . States in other bands are mixed. The  $SU(3)$  decomposition of the lowest bands in  $^{168}\text{Er}$  is shown in Figure 1 and compared to the broken- $SU(3)$  calculations of WCD [3] and CQF [4]. In the PDS calculation the states belonging to the ground and  $\gamma$  bands are the Elliott states  $\phi_E((2N, 0)K=0, L)$

and  $\phi_E((2N-4, 2)K=2, L)$  respectively, while the  $K=0_2$  band is mixed and has the structure

$$|L, K=0_2\rangle = A_1\tilde{\phi}_E((2N-4, 2)\tilde{K}=0, L) + A_2\tilde{\phi}_E((2N-8, 4)\tilde{K}=0, L) \\ + A_3\phi_E((2N-6, 0)K=0, L) . \quad (2)$$

Here  $\tilde{\phi}_E$  denote states orthogonal to the solvable  $\gamma_{K=2k}^k$  Elliott's states. For  $^{168}\text{Er}$  ( $N=16$ ), the  $K=0_2$  band contains 9.6%  $(26, 0)$  and 2.9%  $(24, 4)$  admixtures into the dominant  $(28, 2)$  irrep. Since the geometric analogs of the  $SU(3)$  bands [6] are  $(2N-4, 2)K=0 \sim \beta$ ,  $(2N-8, 4)K=0 \sim (\sqrt{2}\beta^2 + \gamma^2_{K=0})$ ,  $(2N-6, 0)K=0 \sim (\beta^2 - \sqrt{2}\gamma^2_{K=0})$ , it follows that in the PDS calculation, the  $K=0_2$  band contains admixtures of 12.4%  $\gamma^2_{K=0}$  and 0.1%  $\beta^2$  into the  $\beta$  mode. These observations are in line with recent theoretical [7] and experimental [8] claims concerning the importance of a double- $\gamma$  component for the structure of the lowest  $K=0$  excited band.

An important clue to the structure of these collective excitations comes from E2 transitions. The relevant operator is

$$T(E2) = \alpha Q^{(2)} + \theta \Pi^{(2)} \quad (3)$$

where  $Q^{(2)}$  is the quadrupole  $SU(3)$  generator and  $\Pi^{(2)} = (d^\dagger s + s^\dagger \tilde{d})$  is a  $(2, 2)$  tensor under  $SU(3)$ . The parameters  $\alpha$  and  $\theta$  can be extracted from known  $B(E2)$  values for  $2_g^+ \rightarrow 0_g^+$  and  $2_\gamma^+ \rightarrow 0_g^+$  transitions. Since the wave functions of the solvable states are known, it is possible to obtain **analytic** expressions for the E2 rates between them [5]. If we recall that only the ground band has the  $SU(3)$  component  $(\lambda, \mu) = (2N, 0)$ , that  $Q^{(2)}$ , as a generator, cannot connect different  $SU(3)$  irreps and that the  $\Pi^{(2)}$  term can connect the  $(2N, 0)$  irrep only with the  $(2N-4, 2)$  irrep, we obtain the following expressions for  $B(E2)$  values of  $K=0_2 \rightarrow g$  and  $\gamma \rightarrow g$  transitions

$$B(E2; \gamma, L \rightarrow g, L') = \theta^2 \frac{|\langle \phi_E((2N-4, 2)K=2, L) | \Pi^{(2)} | \phi_E((2N, 0)K=0, L) \rangle|^2}{(2L+1)} \\ B(E2; K=0_2, L \rightarrow g, L') = \\ A_1^2 \theta^2 \frac{|\langle \tilde{\phi}_E((2N-4, 2)\tilde{K}=0, L) | \Pi^{(2)} | \phi_E((2N, 0)K=0, L) \rangle|^2}{(2L+1)} \quad (4)$$

The reduced matrix elements of  $\Pi^{(2)}$  are known [9, 10]. It follows that  $B(E2)$  ratios for these transitions are independent of both  $\alpha$  and  $\theta$ . Furthermore, since the ground and  $\gamma$  bands have pure  $SU(3)$  character, the corresponding wave-functions do not depend on parameters of the Hamiltonian and hence are determined solely by symmetry. Consequently, the  $B(E2)$  ratios for  $\gamma \rightarrow g$  transitions are parameter-free predictions of  $SU(3)$  PDS. The latter were shown in [5] to be in excellent agreement with experiment and are similar to those of the WCD calculation [3]. The  $B(E2)$  values for  $K=0_2 \rightarrow g$  transitions are seen from Eq. (4) to be proportional to  $A_1^2$ , hence, in accord with the discussion following Eq. (2), determine the admixture of double-phonon excitations in the  $K=0_2$  band. A comparison with recent data [8] and previous broken- $SU(3)$  calculations [3, 4] is shown in Table 1. The agreement between the PDS predictions and the data confirms the relevance of partial dynamical  $SU(3)$  symmetry to the spectroscopy of  $^{168}\text{Er}$ . This work is supported by a grant from the Israel Science Foundation.

**Table 1.** Comparison of experimental and theoretical absolute B(E2) values [W.u.] for transitions from the  $K = 0_2$  band in  $^{168}\text{Er}$ .

Transition	Exp. [8] B(E2)	range	Calc.		
			PDS	WCD [3]	CQF [4]
$2_{K=0_2}^+ \rightarrow 0_g^+$	0.4	0.06–0.94	0.65	0.15	0.03
$2_{K=0_2}^+ \rightarrow 2_g^+$	0.5	0.07–1.27	1.02	0.24	0.03
$2_{K=0_2}^+ \rightarrow 4_g^+$	2.2	0.4–5.1	2.27	0.50	0.10
$2_{K=0_2}^+ \rightarrow 2_\gamma^+{}^a)$	6.2 (3.1)	1–15 (0.5–7.5)	4.08	4.2	4.53
$2_{K=0_2}^+ \rightarrow 3_\gamma^+{}^a)$	7.2 (3.6)	1–19 (0.5–9.5)	7.52	7.9	12.64

<sup>a)</sup> The two numbers in each entry correspond to an assumption of pure E2 and (in parenthesis) 50% E2 multipolarity.

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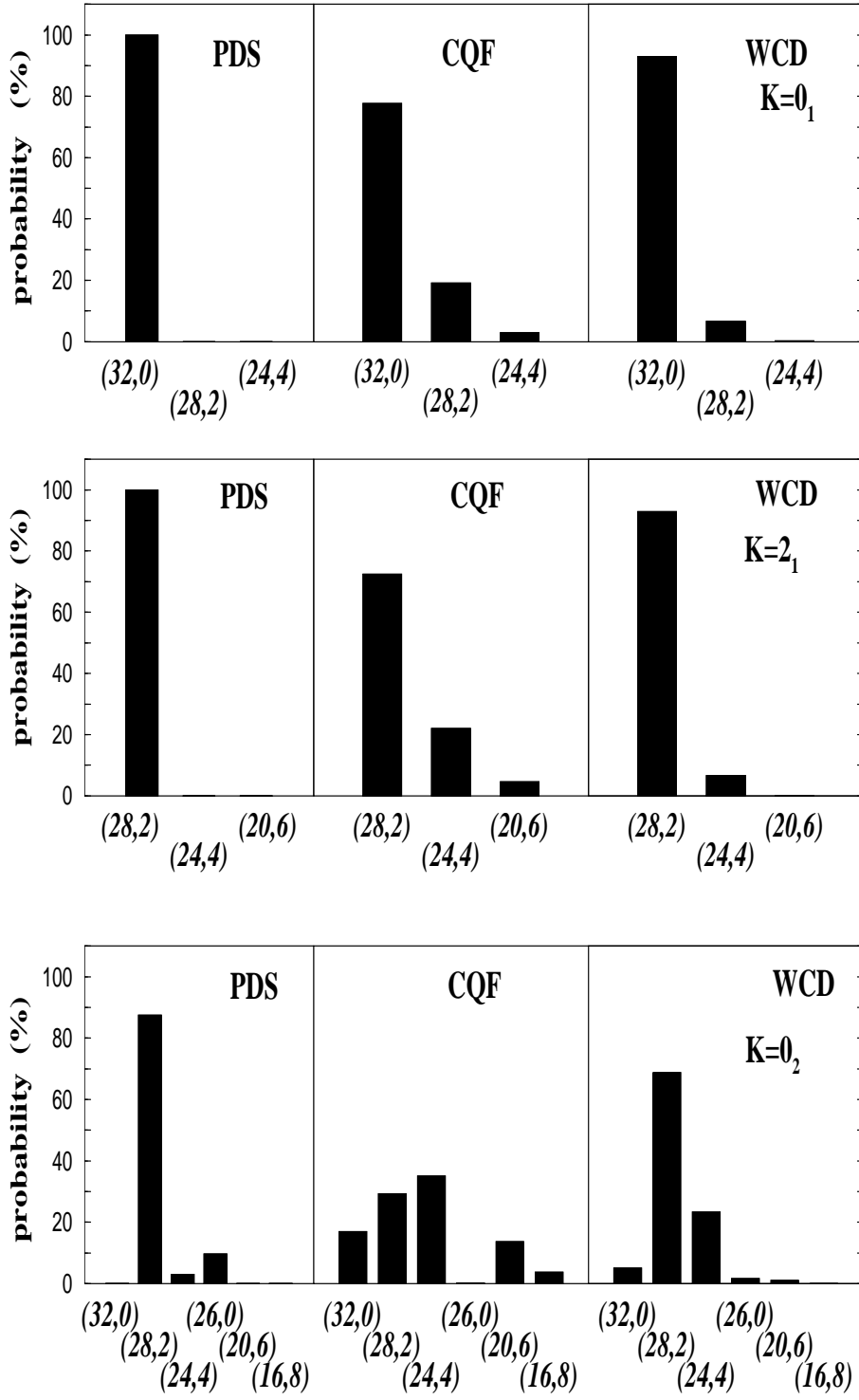


Figure 1:  $SU(3)$  decomposition of wave functions of the ground ( $K = 0_1$ ),  $\gamma$  ( $K = 2_1$ ), and  $K = 0_2$  bands for  $^{168}\text{Er}$  in the PDS (present work), WCD [3] and CQF [4] calculations.